Math 4740 - Homework # 2
Combinatorial Counting

Part 1 - Combinations and Permutations

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1. (a) How many six-digit numbers are there? (b) How many of them contain the digit 5?

2. Suppose that in a state, license plates have 3 letters (from A-Z) followed by 3 numbers (from 0-9) in a way that no letter or number is repeated in a single plate. Determine the number of license plates for this state.

3. (a) How many permutations of the letters a, b, c, d, and e are there?
   (b) How many begin with a and end with c?

4. Let A be the set of all sequences of 0's, 1's, and 2's of length 8.
   (a) How many elements are there in A?
   (b) How many elements of A have exactly four 0's and four 1's?
   (c) How many elements of A have exactly three 0's, three 1's, and two 2's?

Part 2 - Probabilities with dice and coins

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5. Six 6-sided dice are thrown. What is the probability that at least two of them show the same number?

6. Suppose that four 8-sided dice are thrown.
   (a) What is the probability that you will get exactly two 3's?
   (b) What is the probability that you get at most two 8's?
   (c) What is the probability that you get at least three 1's?

7. Suppose that ten 6-sided dice are thrown. Calculate the probability that you will get exactly one 4, exactly six 5's, and the other three numbers are anything other than 4's or 5's.

8. Suppose a coin is tossed 5 times.

   (a) What is the probability that exactly 1 head occurs and the other four tosses are not 1's?

   (b) What is the probability that exactly 3 heads occur?

   (c) What is the probability that all five tosses are tails?

9. Suppose that a coin is tossed 20 times.

   (a) What is the probability that at least 2 heads occurs?

   (b) What is the probability that at most 3 heads occurs?

10. If a six-sided die is rolled four times, what is the probability that a 3 occurs at least once in the four rolls?

11. Suppose that five numbers are selected at random from the numbers

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20$$

    What is the probability that the smallest number selected is larger than 6?

    For example, if you selected the five numbers $6, 10, 2, 19, 20$ then the smallest number 2 would not be larger than 6. However, if you selected the numbers $11, 15, 7, 9, 18$ then the smallest number 7 would be larger than 6.

Part 3 - Probabilities with balls from a bag

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12. Suppose there are 3 green balls and 4 red balls in a bag. You randomly select 2 of the balls from the bag. The order doesn't matter, you grab them all at the same time.

    (a) What is the probability that you picked two red balls?

    (b) What is the probability that you picked one red ball and one green ball?

13. Suppose there balls in a bag labeled 1, 2, 3, 4, 5, 6, 7, 8. You randomly select 2 of the balls from the bag. The order doesn't matter, you grab them all at the same time. What is the probability that you picked two even numbered balls?

Part 4 - Probabilities with SupperLotto and cards

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14. Recall from class that in the CA SupperLotto Plus you pick 5 "lucky" numbers from 1 to 47 (no repeats here and order doesn't matter) and you also pick 1 "mega" number from 1 to 27.

    (a) What is the probability that you get 2 of the 5 lucky numbers correct and the mega number correct?

    (b) What is the probability that you get 4 of the 5 lucky numbers correct and the mega number correct?

15. Suppose you are dealt two cards from a standard 52 card deck.

    (a) What is the probability that they are both aces?

    (b) What is the probability that they both have the same face value? (That is, they are both 3's or both J's.)

    (c) What is the probability that you are dealt blackjack?

    (Blackjack is where one of the cards is an ace and the other card is either a ten, jack, queen, or king. For example, $A\clubsuit$, $10\heartsuit$ is a blackjack.)

16. Suppose you are dealt 5 cards from a standard 52 card deck. Let's calculate the poker probabilities that we didn't do in class.

    (a) What is the probability that you are dealt a flush?

    (b) What is the probability that you are dealt a three of a kind?

17. Suppose from a standard 52-card deck you are dealt five cards. You only know three of the cards and they are $2\clubsuit$, $3\clubsuit$, and $4\clubsuit$. You don't know the other two cards.

(a) What is the probability that the other two cards are both clubs so you have a flush?

(b) What is the probability that the other two cards will give you a straight, but NOT a straight flush?

(c) What is the probability that the other two cards will give you a straight flush?

Part 5 - Compound probability spaces

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18. Suppose we have a 4-sided die labeled $1, 2, 3, 4$, but each side is not equally likely. Suppose that 1 occurs with probability $1/8$, 2 occurs with probability $2/8$, 3 occurs with probability $3/8$, and 4 occurs with probability $2/8$. Suppose we first flip a normal fair coin and then roll this 4-sided die. Create a probability space (S, Ω, P) and draw a tree picture for the probability function.

19. Do the same thing as in the previous problem 18, but instead use a coin where heads occurs with probability 0.7 and tails with probability 0.3.